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Well tailored compressive stress–strain relations for elastomeric foams in uni-axial stress compression

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Well tailored compressive stress–strain relations for elastomeric open and closed cell foams under a uni-axial stress compression were developed. These sets are aimed at replacing those presented by Gibson and Ashby (1988) [1] since they are mismatched and cannot be used. The proposed set of compressible stress–strain relations for elastomeric open cell foams was compared with experimental results. Good agreement was seen.

1. Introduction

In order to simulate phenomena in which flexible foams are compressed as a result of the action of external loads, e.g. shock waves, there is a need to know the compressive stress–strain relations of the foams under consideration.

As shown by Gibson and Ashby [1] and by Ben-Dor and co-workers [2], the compressive stress–strain relations of cellular materials, in general, and foams in particular, depend on the following major factors:

1. the type of foam;
    - elastomeric foams
    - elastic-plastic foams
    - elastic-brittle foams
2. the internal structure of the foam;
    - open cell
    - closed cell
3. the mode of compression;
    - uni-axial stress mode
    - bi-axial stress mode
    - uni-axial strain mode

The general shapes of the various stress–strain relations are similar. They all show linear elastic behaviour at low strains, followed by a long collapse regime in which the stress rises slightly, truncated by a regime of densification in which the stress rises steeply. The main difference between the above mentioned three types of foams is in the mechanical behaviour in their collapse regime. In elastomeric foams the collapse is due to elastic buckling of the cell walls, in elastic–plastic foams the collapse arises from plastic yielding of the cell walls and in elastic–brittle foams the collapse is due to brittle crushing of the cell walls.

A comprehensive and generally accepted book, in which the mechanical properties, in general, and the stress–strain relations, in particular, are derived and given for a variety of cellular materials, has been published by Gibson and Ashby [1]. Chapter 5 of their book is devoted to foams. In section 5.5 they summarize the mechanical behaviour of foams and provide some experimental and theoretical stress–strain maps. Unfortunately, however, when attempting to use their correlations in our numerical simulations, we realized that they are very problematic, and cannot be used. Consequently, it is the aim of this study to develop a correct set of compressive stress–strain relations.

2. Theoretical background

Let us consider, for example, the compressive stress–strain correlations for elastomeric open cell foams under a uni-axial stress mode of compression as proposed by Gibson and Ashby [1] in their equations (5.55a and b). In general they divide the stress–strain plane into two domains, the linear–elastic and the post buckling domains (for more details see [1] pp. 161–162).

In the linear elastic domain ($0 \leqslant \varepsilon \leqslant \varepsilon_{el}$)

$$\sigma = E^* \varepsilon \tag{1}$$

where E^* is the effective modulus of elasticity of the cellular material.

When treating the post-buckling domain they claim that experimental results are "best described in two segments", which they refer to as a plateau and a densification regime. For these regimes they suggested the following two correlations: in the plateau regime ($\varepsilon_{el} \leqslant \varepsilon \leqslant \varepsilon_2$)

$$\sigma = E^* \varepsilon_{el} \tag{2}$$

in the densification regime ($\varepsilon_2 \leqslant \varepsilon < \varepsilon_D$)

$$\sigma = E^* \varepsilon_{el} \frac{1}{D}\left(\frac{\varepsilon_D}{\varepsilon_D - \varepsilon}\right)^m \tag{3}$$

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The last two equations are from equations (5.55a and b) in [1]. In Equations 2 and 3 σ and ε are the stress and the strain of the compressed foam, respectively.

For an open cell foam it can be calculated from

$$E^* = E_s\left(\frac{\rho^*}{\rho_s}\right) \tag{4}$$

where E_s is the modulus of elasticity of the solid material from which the foam is made, ρ^* and ρ_s are the densities of the foam and the solid material from which the foam is made, respectively.

The upper strain limit of the linear elastic regime for an elastomeric open cell foam, ε_{el} can be calculated from either of the two relations

$$\varepsilon_{el} = 0.05 \tag{5.1}$$

or

$$\varepsilon_{el} = 0.03\left[1 + \left(\frac{\rho^*}{\rho_s}\right)^{\frac{1}{2}}\right]^2 \tag{5.2}$$

[Note that these two relations can be simply obtained from equations (5.18a and b) of Gibson and Ashby ([1] p. 140) by inserting into them the definition of E^* as given by their equations (5.6a)].

As discussed by Gibson and Ashby ([1] pp. 139–140), the expression given by (5.2) is a modification of that given by (5.1) for cases when the "density is not small (when cell corners account for a significant part of the volume)". They further state that "the correction is insignificant when $\rho^*/\rho_s < 0.3$".

The upper limit of the plateau regime, ε_2, as suggested by Gibson and Ashby [1] in their equation (5.55a), can be calculated from

$$\varepsilon_2 = \varepsilon_D\left(1 - \frac{1}{D}\right) \tag{6}$$

where, ε_D, which is defined by them as the strain at which complete densification is reached, is given by

$$\varepsilon_D = 1 - 1.4\frac{\rho^*}{\rho_s} \tag{7}$$

In addition, according to Gibson and Ashby ([1] p. 162) D and m, which appear in Equations 3 and 6, are constants for a given class of foams.

Gibson and Ashby ([1] p. 162) claim that based on experimental results (which they present in Fig. 5.34), the values of m and D are: $m = 1$ and $D = 1$ for a polyethylene foam, and $m = 1$ and $D = 1.55$ for polyurethane foam.

Following this statement they drew the stress–strain map for a polyethylene foam (see Fig. 5.33 in their book) while stating "Fig. 5.33 is a mechanism map constructed entirely from the model-based equations", i.e., Equations 1 to 7 in this paper.

As mentioned earlier, our attempts to redraw their theoretical stress–strain map using their model-based equations completely failed. The reasons are presented subsequently.

Interesting $D = 1$ into Equation 6 implies that the upper strain limit of the plateau regime $\varepsilon_2 = 0$. However, for a given foam, for which according to Gibson and Ashby ([1] p.3) $0 < \frac{\rho^*}{\rho_s} \leqslant 0.3$, the lower strain limit of the plateau regime as calculated from Equation 5 results in $\varepsilon_{el} > 0$. Combining the above two facts implies that if $D = 1$ then $\varepsilon_{el} > \varepsilon_2$; hence not only that the lower strain limit of the plateau regime, which is defined in the strain range of $\varepsilon_{el} < \varepsilon < \varepsilon_2$, is larger than its upper strain limit, the lower strain limit of the densification regime, ε_2, i.e. the upper strain limit for the plateau regimes, which for $D = 1$ is equal to zero, is even lower than the upper strain limit of the linear–elastic regime, i.e. the lower strain limit of the plateau regime [which is given by Equation 5]. Hence, there is little doubt that the lower and the upper strain limits of the correlations for the plateau and densification regimes as given by Gibson and Ashby [1] are mismatched, and there is no way to draw the stress–strain map shown in Fig. 5.33 of their book using their correlations.

As mentioned earlier, the values of D and m, both for polyethylene and polyurethane foams, were determined by Gibson and Ashby [1] from experimental results which are shown in Fig. 5.34 of their book. In this figure the experimental results are plotted in the $\left(\log\frac{\sigma}{E^*\varepsilon_{el}}, \log\frac{\varepsilon_D}{\varepsilon_D - \varepsilon}\right)$-plane. Based on this figure they argued that the experimental points both of polyurethane and polyethylene could be best described by straight lines of the form expressed by Equation 3. These straight lines have the general form

$$\log\frac{\sigma}{E^*\varepsilon_{el}} = \log\frac{1}{D} + m\log\frac{\varepsilon_D}{\varepsilon_D - \varepsilon} \tag{8}$$

From the experimental results shown in Fig. 5.34, Gibson and Ashby [1] concluded that the slope, m, of these two lines is $m = 1$. The points where these two lines intersect the horizontal axis can be obtained from

$$0 = \log\frac{1}{D} + m\log\frac{\varepsilon_D}{\varepsilon_D - \varepsilon} \tag{9}$$

which can be rearranged to read

$$D = \left(\frac{\varepsilon_D}{\varepsilon_D - \varepsilon}\right)^m \tag{10}$$

However, since Gibson and Ashby [1] suggested that for both lines $m = 1$, the value of D can be simply obtained from the relation

$$D = \frac{\varepsilon_D}{\varepsilon_D - \varepsilon} \tag{11}$$

Following this analysis, Gibson & Ashby [1] obtained the above quoted values for polyethylene ($m = 1$ and $D = 1$) and for polyurethane foams ($m = 1$ and $D = 1.55$).

However, as shown earlier the values appropriate to polyethylene, i.e. $m = 1$ and $D = 1$ are problematic. Since Gibson and Ashby [1] deduced these values from experimental results which are presented in their book, a re-evaluation of their experimental result was a natural step.

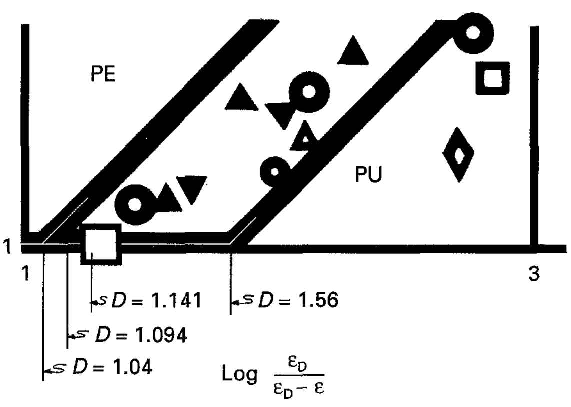

Figure 1 Reproduction of the lower left part of Fig. 5.33 from, Gibson and Ashby [1].

Fig. 1 is an enlargement of the left hand side corner of Gibson and Ashby's original Fig. 5.34 [1]. As can be seen the line appropriate to a polyethylene foam (PE) does not intersect the horizontal axis at the origin and hence there is little doubt that based on their own drawing, $D \neq 1$ for a polyethylene foam. Characterizing the wide black lines shown in Fig. 1 by thin white centre lines, indicates that the value of D for a polyethylene foam is $D = 1.04$. Although, very close to unity, assigning an approximate value of $D = 1$ to polyethylene (as was probably done by Gibson and Ashby [1]) introduces an unavoidable singularity in the upper strain limit of the plateau regime, ε_2, (see Equation 6) and as a result a mismatch in the lower and upper strain limits of the plateau regime.

Furthermore, as can be seen from the strain range of the plateau regime, the lower strain limit, ε_2, must be greater than the upper strain limit, ε_{el}. In terms of Equations 5.1, 6 and 7 this means that for an open cell foam

$$\left(1 - 1.4\frac{\rho^*}{\rho_s}\right)\left(1 - \frac{1}{D}\right) \geqslant 0.05 \qquad (12)$$

The largest value of $\frac{\rho^*}{\rho_s}$ according to Gibson and Ashby ([1] p.3) for which their model and analysis of cellular materials is still valid, is 0.3. Inserting this value into the inequality given by Equation 12 results in

$$D \geqslant 1.094 \qquad (13)$$

Note that replacing the right-hand side in the inequality given by Equation 13, by the strain limit of the linear elastic regime which includes density corrections, i.e. Equation 5.2 instead of 5.1 would result in $D \geqslant 1.141$. The points along the $\log\frac{\varepsilon_D}{\varepsilon_D - \varepsilon}$ axis which correspond to $D = 1.094$ and 1.141 are indicated on Fig. 1.

Shifting the line appropriate to a polyethylene foam (PE) such that it will intersect the $\log\frac{\varepsilon_D}{\varepsilon_D - \varepsilon}$ axis at either of the points where $D = 1.094$ or 1.141 while keeping it parallel to the original line, i.e. $m = 1$, results in a situation in which the shifted line does not seem to be an adequate fit, since almost all the experimental points lie to the left of the shifted line.

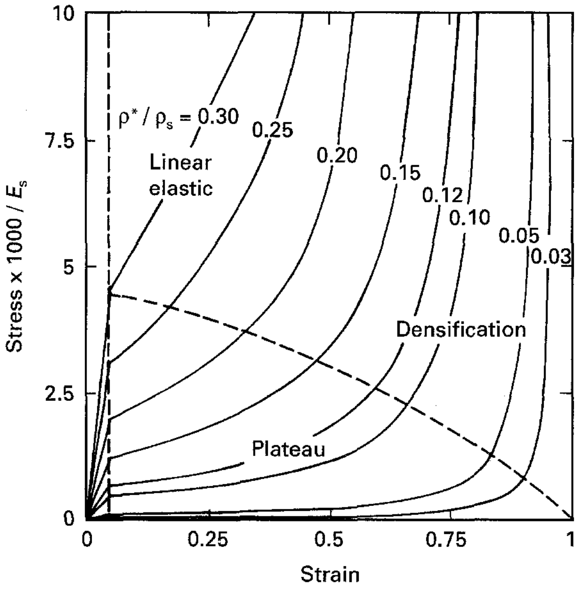

Figure 2 The "theoretical" stress–strain map as given in Fig. 5.33 of Gibson and Ashby [1].

In addition, an inspection of Fig. 2, which is a reproduction of Gibson and Ashby's "theoretical" stress–strain map (Fig. 5.33 in [1]), and which according to them was "constructed entirely from the model based equations", reveals that all the $\sigma - \varepsilon$ curves in the plateau regime are far from being horizontal as they should have been had they been actually drawn using Equation 2 [equation (5.55a)] which simply implies that σ = constant inside the plateau regime.

Furthermore, the values of ε_D, i.e. the strain at complete densification, as calculated from Equation 7 (equation (5.22) in Gibson and Ashby [1]) for the relative densities, ρ^*/ρ_s, for which the $\sigma - \varepsilon$ curves are plotted in Fig. 2, do not appear to be the values of ε_D which are evaluated from Fig. 2. For example, while Equation 6 yields for $\rho^*/\rho_s = 0.03$ a value of $\varepsilon_D = 0.958$, the corresponding value of ε_D as it appears in Fig. 2 is larger than 0.961.

The foregoing discussion clearly indicates that, unfortunately, the set of the semiempirical compressive stress–strain relations for an elastomeric open cell polyethylene foam under uni-axial compressive stress as given by Gibson and Ashby [1] is wrong. We would also like to point out that based on the experimental results which are shown in Fig. 3, which is a reproduction of Fig. 5.32 of Gibson and Ashby's book, the actual $\sigma - \varepsilon$ curves never behave as a perfect plateau. Hence, imposing an expression describing a perfect plateau, as implied by Equation 2, is artificial and redundant.

Finally, the correlations of Gibson and Ashby [1] are limited to strains smaller than the strain at which complete densification is reached, i.e. for $\varepsilon < \varepsilon_D$, since according to their semi-empirical relation (Equation 3) $\sigma \rightarrow \infty$ when $\varepsilon \rightarrow \varepsilon_D$. However, according to Gibson and Ashby ([1] p. 142) when densification is completed, i.e. when $\varepsilon \rightarrow \varepsilon_D$, "all the pore space has

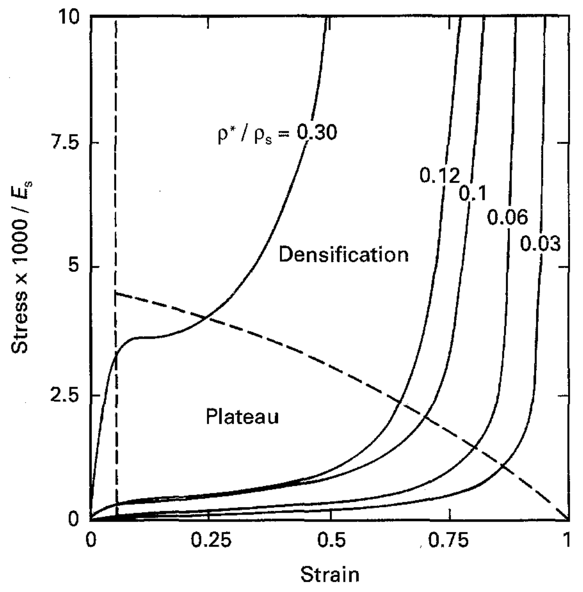

Figure 3 The experimental stress–strain map as given by Fig. 5.32 of Gibson and Ashby [1].

been squeezed out", This implies that the cellular material practically transforms to a non-cellular material. For this reason, "when this happens the stress–strain curve rises steeply, tending to a slope of E_s". Practically, this means that further compression after the strain, ε, reaches the complete densification strain, ε_D, should be possible, i.e. the stress σ cannot approach infinity at $\varepsilon \rightarrow \varepsilon_D$. Instead, the densified material should resemble a linear–elastic behaviour of the solid material from which the foam is made. Due to the increasing interest in the head-on collision of shock waves with cellular materials, e.g. Gvozdeva and colleagues [3] Korobeinikov and Utriew [4], Henderson et al. [5], Skews et al. [6], Mazor et al. [7] and Ben-Dor et al. [2], there is a need for a set of stress–strain relations which will enable the strain to exceed ε_D, and enable a continuous transition from a cellular material behaviour to a non-cellular solid material behaviour when $\varepsilon = \varepsilon_D$ is reached.

As a result of the foregoing discussion it was decided to follow the suggestions and ideas of Gibson and Ashby [1] and develop a new set of empirical compressive stress–strain relations which will be well tailored at their lower and upper strain limits. The set will cover the uni-axial stress mode of compression both for open and closed cell elastomeric foams. Since it will enable reaching strains beyond the strain at which complete densification is reached, it could be also used by investigators interested in simulating phenomena in which solid foams are compressed by relatively large external forces.

3. Present study

As mentioned earlier the compressive stress–strain relations of a foam depends on the following three factors:

1. the type of the foam, i.e. elastomeric, elastic–plastic or elastic–brittle foam,
2. the internal structure of the foam, i.e. open or closed cell, and
3. the loading mode, i.e. uni-axial stress, bi-axial stress or uni-axial strain mode of compression.

In the following we propose two sets of compressive stress–strain relations for elastomeric open and closed cell foams which undergo uni-axial stress compression. Development of compressive stress–strain relations for other modes of compression is the subject of future study.

As will be shown subsequently, a comparison of our proposed set of compressive stress–strain relations with typical results taken from Gibson and Ashby's [1] experimental stress–strain map for the case of a uni-axial stress mode of compression, indicates that our proposed set reproduces the experimental results very well.

The detailed derivation of the sets of the stress–strain relations are not given in the following since they can be deduced from [1]. However, since Gibson and Ashby did not refer to the compression after complete densification is reached, i.e. the domain which is reached when $\varepsilon \geqslant \varepsilon_D$, the derivation of the compressive stress–strain relations for this regime is given in Appendix A.

3.1. Elastomeric open cell foams

Basically we adopt Gibson and Ashby's [1] approach and divide the (σ,ε)-plane into two domains, namely the linear–elastic and the post-buckling domains.

In the linear–elastic regime $(0 \leqslant \varepsilon \leqslant \varepsilon_{el})$ the classical relation, which was given earlier as Equation 1 is adopted $(\sigma = E^* \varepsilon)$. E^*, the effective modulus of elasticity of the open cell elastomeric foam is given by Equation 4. However, unlike Gibson and Ashby [1] who subdivided the post-buckling domain into two regimes, namely the plateau and the densification regimes, and accordingly proposed different expressions for each of these regimes [Equations 2 and 3, respectively], we propose only one expression for the entire post-buckling domain. The reason for adopting this approach lies in the earlier mentioned fact that in reality (see the experimental results in Fig. 2 which is a reproduction of Fig. 5.32 of Gibson and Ashby [1]) a real plateau behaviour does not exist. Hence, instead of forcing an "artificial" constraint on the stress–strain relations, we propose an expression which allows a slight increase in stress with increasing strain in the regime which is referred to by Gibson and Ashby [1] as a plateau. The upper strain limit of this expression is the strain at complete densification, ε_D, which has already been defined in Equation 7. Consequently, in the post-buckling regime $(\varepsilon_{el} \leqslant \varepsilon \leqslant \varepsilon_D)$ we propose

$$\sigma = \sigma_{el} \frac{\varepsilon_D}{\varepsilon_D + \varepsilon_{el} - \varepsilon} \tag{14}$$

σ_{el} is the stress at the upper strain limit of the linear elastic domain. It can be simply obtained from Equation 13 by setting $\varepsilon = \varepsilon_{el}$ where ε_{el} is given by Equation 5.1 or 5.2. Note that unlike Gibson and Ashby's correlations (Equations 2 and 3) which are limited to

$\varepsilon < \varepsilon_D$, Equation 14 indicates that the strain can actually reach the strain at complete densification, ε_D.

Once the strain ε reaches the complete densification strain ε_D, the foam practically ceases to exist since its pores vanish and it transforms to a solid material whose mechanical properties are identical to those of the solid material from which the foam is made. Since this occurs for the strain range $\varepsilon \geqslant \varepsilon_D$, we refer to this domain as the *post-complete-densification domain.* Based on the detailed derivation given in Appendix A, one obtains in the post-complete-densification regime ($\varepsilon_D \leqslant \varepsilon < 1$)

$$\sigma = \sigma_{el}\frac{\varepsilon_D}{\varepsilon_{el}} + E_s\frac{\varepsilon - \varepsilon_D}{1 - \varepsilon_D} \qquad (15)$$

where E_s is the modulus of elasticity of the solid material from which the foam is made.

3.2. Elastomeric closed cell foams

Similarly to the compressive stress–strain relations for elastomeric open cell foams, we again divide the (σ,ε)-plane into three domains, namely; the linear–elastic, the post buckling and the post complete densification domains. For the case of a closed cell elastomeric foam under a uni-axial stress compression, we proposed the following set of compressive stress–strain relations. In the linear–elastic regime ($0 \leqslant \varepsilon \leqslant \varepsilon_{el}$) $\sigma = E^* \varepsilon$. In the post-buckling regime ($\varepsilon_{el} \leqslant \varepsilon \leqslant \varepsilon_D$)

$$\sigma = \sigma_{el} + p_0\frac{(\varepsilon - \varepsilon_{el})\left(1 - \dfrac{\rho^*}{\rho_s}\right)}{\left(1 - \varepsilon - \dfrac{\rho^*}{\rho_s}\right)\left(1 - \varepsilon_{el} - \dfrac{\rho^*}{\rho_s}\right)} \qquad (16)$$

where p_0 is the initial pressure in the closed pores. In the post complete densification regime ($\varepsilon_D \leqslant \varepsilon < 1$)

$$\sigma = \sigma_{el} + p_0\frac{(\varepsilon_D - \varepsilon_{el})\left(1 - \dfrac{\rho^*}{\rho_s}\right)}{\left(1 - \varepsilon_D - \dfrac{\rho^*}{\rho_s}\right)\left(1 - \varepsilon_{el} - \dfrac{\rho^*}{\rho_s}\right)} + E_s\frac{\varepsilon - \varepsilon_D}{1 - \varepsilon_D} \qquad (17)$$

where E^* the effective modulus of elasticity of an elastomeric closed cell foam, is given by

$$E^* = E_s\left[\varphi^2\left(\frac{\rho^*}{\rho_s}\right)^2 + (1 - \varphi)\frac{\rho^*}{\rho_s} + \frac{p_0}{E_s}\frac{1 - 2\upsilon^*}{1 - \rho^*/\rho_s}\right] \qquad (18)$$

here φ is the fraction of solid material in the cell edges. From Fig. 5.9 in Gibson and Ashby [1] it is clear that $0.6 \leqslant \varphi \leqslant 0.8$, and υ^* is the Poisson's ratio of the foam. The upper strain limit of the linear elastic regime for an elastomeric closed cell foam, ε_{el}, can be calculated from either of the two relations

$$\varepsilon_{el} = \frac{0.05\left(\dfrac{\rho^*}{\rho_s}\right)^2 + \dfrac{p_0 - p_{at}}{E_s}}{\varphi^2\left(\dfrac{\rho^*}{\rho_s}\right)^2 + (1 - \varphi)\dfrac{\rho^*}{\rho_s} + \dfrac{p_0(1 - 2\upsilon^*)}{E_s(1 - \rho^*/\rho_s)}} \qquad (19.1)$$

or

$$\varepsilon_{el} = \frac{0.03\left(\dfrac{\rho^*}{\rho_s}\right)^2\left[1 + \left(\dfrac{\rho^*}{\rho_s}\right)^{\frac{1}{2}}\right]^2 + \dfrac{p_0 - p_{atm}}{E_s}}{\varphi^2\left(\dfrac{\rho^*}{\rho_s}\right)^2 + (1 - \varphi)\dfrac{\rho^*}{\rho_s} + \dfrac{p_0(1 - 2\upsilon^*)}{E_s(1 - \rho^*/\rho_s)}} \qquad (19.2)$$

where p_{atm} is the ambient pressure. The strain at complete densification, ε_D, is given by Equation 7. (These relations can be simply obtained by dividing Equations 5.21a and b) by Equation (5.13a) of Gibson and Ashby [1]).

Similar to the case of an open cell elastomeric foam, here again Gibson and Ashby provided two expressions of ε_{el} where the second one (Equation 19.2) is a modification of the first one (Equation 19.1) when density corrections are accounted for. Recall that as claimed by Gibson and Ashby ([1] p. 139). "the correction is insignificant when $\rho^*/\rho_s < 0.3$".

4. Results and discussion

The validity of any set of empirical stress–strain relations depends on its capability of reproducing actual experimental results.

A comparison between the experimental results (solid lines), the "theoretical" results of Gibson and Ashby (dashed-dotted lines) and the results of the model proposed in section 3.1 (dashed line) for an elastomeric open cell foam under a uni-axial stress compression is shown in Fig. 4, for three different values (small, moderate and large) of relative densities: $\rho^*/\rho_s = 0.03$, 0.1 and 0.3. As mentioned earlier, our attempts to use Gibson and Ashby's correlations [1], which are given by Equations 1 to 7, in order to reproduce their "theoretical" stress–strain map for $D = 1$ and $m = 1$, which is shown in Fig. 2, have failed. Hence, the curves related to them in Fig. 4 are simply copied from Fig. 5.33 of their book which is referred by them as a theoretical stress–strain map.

In their theoretical stress–strain map (see Fig. 2), they used for the upper strain limit of the linear elastic regime, ε_{el}, the value given by Equation 5.1, i.e. $\varepsilon_{el} = 0.05$ which, unlike the one given by Equation 5.2, does not include a density correction.

It is evident from Fig. 4 that our correlations reproduce better the experimental results than the curves of Gibson and Ashby [1] at large density ratios, i.e. $\rho^*/\rho_s = 0.3$. At moderate density ratios, i.e. $\rho^*/\rho_s = 0.1$, the correlations proposed by us resemble a better agreement in the strain range appropriate to the regime which is referred to by Gibson and Ashby as the plateau regime. Beyond this strain range, i.e. in the strain range appropriate to the regime which is

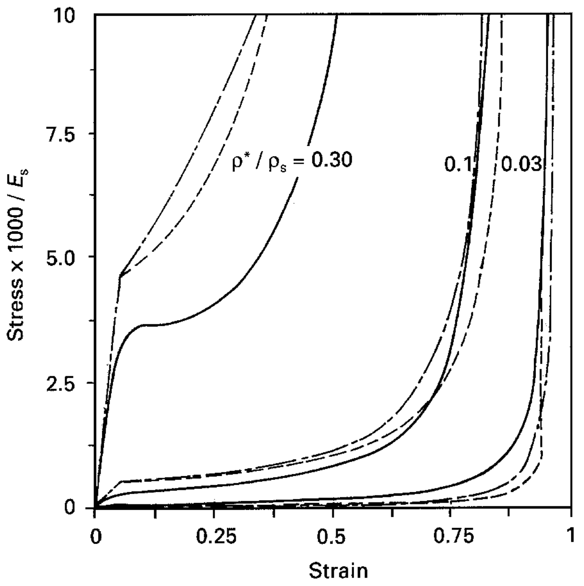

Figure 4 Comparison between theoretical models and experimental results: (——); experimental results; (— — — —), Gibson and Ashby's model; (– – – –) present model.

referred to by Gibson and Ashby as the densification regime, their curves agree better with the experimental results than the curves calculated using the proposed correlations. At low density ratios, i.e. $\rho^*/\rho_s = 0.03$ both Gibson and Ashby's curves and our correlations are seen to be quite similar in the so-called plateau regime. Beyond this regime, in the so-called densification regime, our correlations are much better than the curves of Gibson and Ashby. As a matter of fact the curves predicted by our correlations exactly coincide with the experimental results. It should be stressed here that although we conduct a comparison between Gibson and Ashby's "theoretical" curves and the curves which result from our proposed correlations, one should recall that Gibson and Ashby's curves were not calculated by their correlations which, as mentioned earlier, are mismatched and cannot be used.

Based on the comparison shown in Fig. 4, it can be concluded that not only are the correlations proposed by us capable of reproducing well enough the experimental results in the case of a uni-axial stress compression of an open cell elastomeric foam, they actually resemble a better agreement with the experimental results than the curves of Gibson and Ashby [1].

5. Conclusions

Based on Gibson and Ashby [1] two sets of stress–strain relations for open and closed cell elastomeric foams, have been developed.

The stress–strain relations for an elastomeric open cell foam under a uni-axial stress mode of compression was found to be in good agreement with experimental results which were provided by [1]. Unlike the empirical set given by Gibson and Ashby [1], which as shown in this paper are very problematic as they are mismatched, the presently developed sets are well tailored at their lower and upper strain limits. Moreover, while Gibson and Ashby's model is limited to the strain at which complete densification is reached, $\varepsilon < \varepsilon_D$, the relations developed throughout the course of this study enable a continuous transition from a cellular material behaviour to that appropriate to a non-cellular material of the solid from which the cellular material is made.

Finally, it is worth noting here that the density ratios of the foams used by Ben-Dor et al. [2], Gvozdeva et al. [3] and Skews et al. [6] was $0.02 < \rho^*/\rho_s < 0.05$. These density ratios are typical of foams used by many other researchers who investigate the interaction of shock waves with porous materials. As shown in Fig. 4, this is the range where the correlations proposed in this paper appear to agree very well with the experimental results.

Appendix. The stress–strain relation in the post-complete densification domain

Fig. A.1 is a schematic illustration of the cellular material which has been compressed to its complete densification, i.e. the cellular material which originally had an initial length of L_{c0} was shortened by $L_{c0}\varepsilon_D$ and reached a new length, L_{s0}, equal to

$$L_{s0} = L_{c0}(1 - \varepsilon_D) \tag{A1}$$

At this stage all the pores have collapsed and as a consequence if compression continues one has to deal with a new situation in which a solid material (from which the cellular material was made) is compressed from an initial length L_{s0} and an initial stress σ_{s0}. The strain, ε_s, in this domain is simply defined as

$$\varepsilon_s = \frac{\partial \xi_s}{\partial L_{s0}} \tag{A2}$$

where ξ_s is the displacement of the solid particles in the post complete densification domain.

Based on the foregoing presentation the stress, σ, in the solid material can be simply expressed as

$$\sigma = \sigma_{s0} + E_s\varepsilon_s \tag{A3}$$

where the initial stress, σ_{s0} for open cell elastomeric foams can be obtained from Equation 15 by setting $\varepsilon = \varepsilon_D$

$$\sigma_{s0} = \sigma_{el}\frac{\varepsilon_D}{\varepsilon_{el}} \tag{A4}$$

For closed cell elastomeric foams, the initial stress can be obtained in a similar way from Equation 17

$$\sigma_{s0} = \sigma_{el} + p_0\frac{(\varepsilon_D - \varepsilon_{el})\left(1 - \frac{\rho^*}{\rho_s}\right)}{\left(1 - \varepsilon_D - \frac{\rho^*}{\rho_s}\right)\left(1 - \varepsilon_{el} - \frac{\rho^*}{\rho_s}\right)} \tag{A5}$$

Note that all the parameters which appear in Equations A4 and A5 have been defined in the main text.

In order to express Equation A3 in terms of the stress of the cellular material, ε, which is defined,

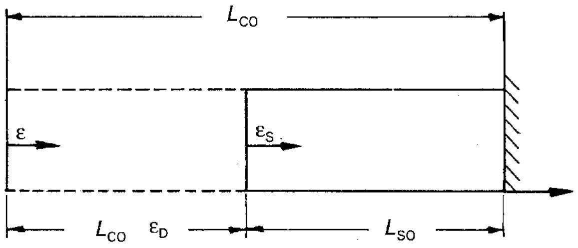

Figure A1 Definition of various parameters.

similarly to Equation A2, as

$$\varepsilon = \frac{\partial \xi_c}{\partial L_{c0}} \tag{A6}$$

where ξ_c is the displacement of the cellular material, where

$$\xi_c = \xi_s + L_{c0}\varepsilon_D \tag{A7}$$

Inserting Equation A7 into equation A2 yields

$$\varepsilon_s = \frac{\partial \xi_c}{\partial L_{s0}} - \frac{\partial L_{c0}\varepsilon_D}{\partial L_{s0}} \tag{A8}$$

Together with Equation A1, Equation A8 becomes

$$\varepsilon_s = \frac{\partial \xi_c}{\partial L_{s0}} - \frac{\varepsilon_D}{1 - \varepsilon_D} \tag{A9}$$

However, Equations A1 and A6 could be combined to

$$\frac{\partial \xi_c}{\partial L_{s0}} = \frac{\partial \xi_c}{\partial L_{c0}}\frac{\partial L_{c0}}{\partial L_{s0}} = \frac{\partial \xi_c}{\partial L_{c0}}\frac{1}{1 - \varepsilon_D} \tag{A10}$$

which together with Equation A6 results in

$$\frac{\partial \xi_c}{\partial L_{s0}} = \frac{\varepsilon}{1 - \varepsilon_D} \tag{A11}$$

Combining equations A9 and A11 results in

$$\varepsilon_s = \frac{\varepsilon - \varepsilon_D}{1 - \varepsilon_D} \tag{A12}$$

Finally, inserting Equation A12 into Equation A3 results in

$$\sigma = \sigma_{s0} + E_s\frac{\varepsilon - \varepsilon_D}{1 - \varepsilon_D} \tag{A13}$$

Equation 11 for an open-cell foam and 17 for a closed cell foam are simply derived from Equation A13.

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